Math 2X03 - Homework 2

Due: May 17, 2016

Chapters Covered on Homework - 12.6, 14.2-14.4, 15.4-15.7

1. Use traces to sketch and identify the surfaces

   (a) (Chapter 12.6 # 11) $x = y^2 + 4z^2$ (b) $16x^2 = y^2 + 4z^2$

2. Find the limit, if it exists, or show that the limit does not exist.

   (a) (Chapter 14.2 # 17) $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$

   (b) (Chapter 14.2 # 18) $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

3. Find the first partial derivatives of the following functions:

   (a) (Chapter 14.3 # 33) $w = \ln(x + 2y + 3z)$

   (b) (Chapter 14.3 # 38) $\phi(x, y, z, t) = \frac{\alpha x + \beta y^2}{\gamma z + \delta t^2}$

4. (Chapter 14.4 #4) Find the equation of the tangent plane to the surface $z = xe^{xy}$ at $(2, 0, 2)$.

5. (Chapter 15.4 #27) Use polar (or cylindrical) coordinates to find the volume of the solid inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$.

6. (Chapter 15.4 #30) Evaluated the iterated integral by converting to polar coordinates

$$\int_0^a \int_{-\sqrt{a-y^2}}^0 x^2 y \, dx \, dy$$

7. (Chapter 15.5 #6) Find the center of mass of the lamina that occupies the triangular region D enclosed by the lines $x = 0$, $y = x$, $2x + y = 6$ and density function $\rho(x, y) = x^2$.

8. (Chapter 15.5 #28) Suppose X and Y are random variables with joint density function

$$f(x,y) = \begin{cases} 0.1e^{-(0.5x+0.2y)} & \text{if } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Verify that f is indeed a joint density function.

   (b) Find the following probabilities

      i. $P(Y \geq 1)$

ii. $P(X \leq 2, Y \leq 4)$

(c) Find the expected values of X and Y. (Set up the integrals but do not evaluate)

9. Find the area of the following surfaces:

   (a) (Chapter 15.6 # 7) The part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$

   (b) The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies within the cylinder $x^2 + y^2 = 2x$ and above the xy-plane.

10. Use triple integrals to find the volume of solid enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$.

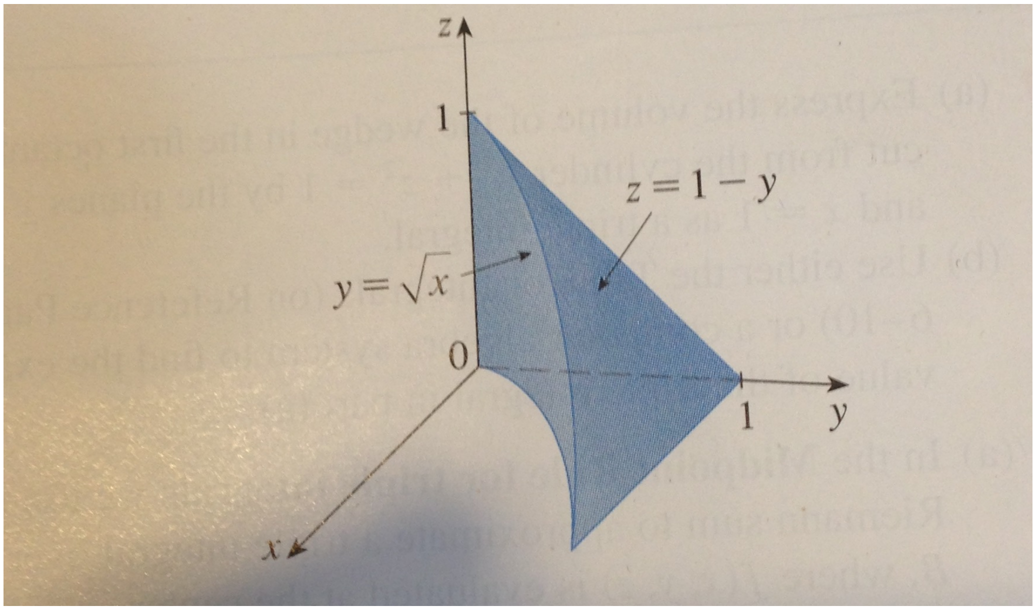
11. Evaluate the following triple integrals:

    (a) (Chapter 15.7 #15)
    $$\iiint_T x^2 \, dV,$$
    where T is the solid tetrahedron with vertices $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$.

    (b) (Chapter 15.7 #18)
    $$\iiint_E z \, dV,$$
    where E is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$ in the first octant.

12. (Chapter 15.7 # 40) Find the mass and the center of mass of the solid E bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $x + z = 1$, $x = 0$, and $z = 0$ with density function $\rho(x, y, z) = 4$.

13. (Chapter 15.7 #33) The figure on the next page shows the region of integration for the integral
    $$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) \, dz \, dy \, dx$$
    Rewrite the integrals as an equivalent iterated integral in five other orders.


z
1
z = 1 − y
y = √x
0
1
y
x